

# Scalar resonance effects on the $B_s - \bar{B}_s$ mixing angle

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The  $B_s^0 \rightarrow J/\psi\phi$  and  $B_s^0 \rightarrow J/\psi f_0(980)$  decays are analyzed within generalized QCD factorization including all leading-order corrections in  $\alpha_s$ . We point out that the ratio of our calculated widths,  $\Gamma(B_s^0 \rightarrow J/\psi f_0(980), f_0(980) \rightarrow \pi^+\pi^-)/\Gamma(B_s^0 \rightarrow J/\psi\phi, \phi \rightarrow K^+K^-)$ , strongly indicates that  $S$ -wave effects in the  $f_0(980)$ 's daughter pions or kaons cannot be ignored in the extraction of the  $B_s - \bar{B}_s$  mixing angle,  $-2\beta_s$ , from the  $B_s^0 \rightarrow \phi J/\psi$  decay amplitudes.

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## I. INTRODUCTION

In the Standard Model,  $CP$  violation is predicted in weak decays thanks to the single phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. It is also well known that such a weak phase is not sufficient to generate a  $CP$  violating decay amplitude. Strong phases are necessary and their strength may significantly enhance the effect of the weak phase. Therefore, hadronic effects, such as resonances of daughter particles in  $S$ - and higher waves, require a careful analysis in the determination of  $CP$  violating phases in hadronic two- and three-body decays [1–4].

The antimatter-matter asymmetry is expected to be very small in weak decays of  $B_s$  mesons; any observed deviation may well be a signal of physics whose origins lie beyond the Standard Model. In the  $B_s^0 \rightarrow J/\psi\phi$  channel, recent measurements by the CDF [5] and DØ [6, 7] Collaborations of the  $B_s - \bar{B}_s$  mixing phase,  $-2\beta_s$ , while not definitive, are considerably larger than Standard Model predictions. Taking advantage of the fact that the  $B_s^0 \rightarrow J/\psi f_0(980)$  channel does not require any angular analysis, one can compute the ratio between the  $B_s^0 \rightarrow J/\psi\phi$  and  $B_s^0 \rightarrow J/\psi f_0(980)$  decay widths in order to estimate the  $\pi^+\pi^-$   $S$ -wave effect on the value of  $\beta_s$ . A first qualitative attempt to predict the ratio,

$$\mathcal{R}_{f_0/\phi} = \frac{\Gamma(B_s^0 \rightarrow J/\psi f_0(980), f_0(980) \rightarrow \pi^+\pi^-)}{\Gamma(B_s^0 \rightarrow J/\psi\phi, \phi \rightarrow K^+K^-)}, \quad (1)$$

was made by Stone and Zhang [8] and gives a result of

the order of 20% – 30%. Their estimate relies on experimental data on  $D_s^+ \rightarrow f_0(980)\pi^+$  and  $D_s^+ \rightarrow \phi\pi^+$  decays and seems to indicate that the  $S$ -wave contribution of  $f_0(980) \rightarrow K^+K^-$  cannot be ignored when analyzing the angle  $\beta_s$  in  $B_s^0 \rightarrow J/\psi\phi$ . Likewise, Xie *et al.* found the effect of an  $S$ -wave component on  $2\beta_s$  to be of the order of 10% in the  $\phi$  resonance region [9].

Based on the QCD factorization (QCDF) formalism we perform a first robust calculation of the ratio  $\mathcal{R}_{f_0/\phi}$ . To this end, all the available observables (polarizations and branching ratio in  $B_s^0 \rightarrow J/\psi\phi$ ) are used to effectively constrain the analysis of the  $B_s^0 \rightarrow J/\psi\phi$  channel. The branching ratio and  $CP$  asymmetry are then predicted for  $B_s^0 \rightarrow J/\psi f_0(980)$ , where we assume that merely the  $s\bar{s}$  component of the  $f_0(980)$  is involved in the hadronic  $B_s \rightarrow f_0(980)$  transition matrix element.

In Section II we introduce the general expressions for the  $B_s^0 \rightarrow J/\psi\phi$  and  $B_s^0 \rightarrow J/\psi f_0(980)$  weak decay amplitudes whereas Sections III and IV provide the details on the leading order corrections in  $\alpha_s$  for both these amplitudes, respectively. In Section V, we list all numerical values of input parameters and briefly recall our model for the  $B_s \rightarrow f_0(980)$  transition form factor [10] on which the ratio  $\mathcal{R}_{f_0/\phi}$  directly depends; we also define the parametrization for the  $B_s \rightarrow \phi$  form factor. Section VI is devoted to our results and, finally, conclusions are drawn in Section VII.

## II. GENERAL FORM OF THE $B_s^0 \rightarrow \phi J/\psi$ AND $B_s^0 \rightarrow f_0(980)J/\psi$ DECAY AMPLITUDES

It is important to realize beforehand that the application of QCDF, following Refs. [11–14], to  $B_s^0$  decays into a heavy-light final state is not self-evident. In both final states,  $\phi J/\psi$  and  $f_0(980)J/\psi$ , the  $s$ -spectator quark is absorbed by the light meson while the emitted meson is heavy, in which case QCDF is not reliable [11]. Nonetheless, as argued in Refs. [15, 16] and more re-

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cently in Ref. [17], the production of a heavy charmonium  $\bar{q}q$  pair bears “color transparency” properties similar to those of a light meson, provided this color-singlet pair is small compared to the inverse strong interaction scale,  $1/\Lambda_{\text{QCD}}$ . This was explicitly demonstrated in next-to-leading order calculations for exclusive  $B$  decays to  $J/\psi$  final states ( $J/\psi K, J/\psi K^*$ ), where infrared divergences

were shown to cancel [15, 16].

In the following, we present the  $B_s^0$  decay amplitudes in which the short- and long-distance contributions are factorized in the approximation of a quasi two-body state,  $M_1 M_2$ , where either  $M_1 M_2 = f_0(980)J/\psi$  or  $M_1 M_2 = \phi J/\psi$ . We begin with the  $B_s^0 \rightarrow \phi J/\psi$  amplitude which can be written for each helicity,  $h = -1, 0, 1$ , as [14],

$$\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^h = \sum_{q=u,c} \lambda_q \left\{ A_{\phi J/\psi}^h \left[ \delta_{qc} \left( a_2^{q,h}(m_b) + \zeta^h \right) + a_3^{q,h}(m_b) + a_5^{q,h}(m_b) + a_7^{q,h}(m_b) + a_9^{q,h}(m_b) \right] \right\}_{\phi J/\psi}. \quad (2)$$

Summing over all the possible helicities, the squared modulus of the total amplitude reads

$$|\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}|^2 = |\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{h=-1}|^2 + |\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{h=0}|^2 + |\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{h=+1}|^2. \quad (3)$$

The  $\bar{B}_s^0 \rightarrow \phi J/\psi$  decay amplitude is obtained by exchange of helicity signs,  $h = +1 \rightarrow h = -1$ , and replacing  $\lambda_q$  by its complex conjugate. The  $B_s^0 \rightarrow f_0(980)J/\psi$  amplitude is,

$$\mathcal{A}_{B_s^0 \rightarrow f_0 J/\psi} = \sum_{q=u,c} \lambda_q \left\{ A_{f_0 J/\psi} \left[ \delta_{qc} \left( a_2^q(m_b) + \zeta \right) + a_3^q(m_b) + a_5^q(m_b) + a_7^q(m_b) + a_9^q(m_b) \right] \right\}_{f_0 J/\psi}. \quad (4)$$

The different elements entering in the amplitudes (2) and (4) are defined in Eqs. (6), (7), (15), (22) and (24). The  $CP$  conjugate  $\bar{B}_s^0$  decay amplitude is again found by replacing  $\lambda_q$  by its complex conjugate.

With the generic amplitude,  $\mathcal{A}_{B_s^0 \rightarrow M_1 J/\psi}$ , the branching ratio,

$$\mathcal{B}(B_s^0 \rightarrow M_1 J/\psi) = \frac{1}{\Gamma_{B_s^0}} \frac{1}{16\pi m_{B_s^0}} \times \lambda^{1/2} \left( 1, m_{M_1}^2/m_{B_s^0}^2, m_{J/\psi}^2/m_{B_s^0}^2 \right) |\mathcal{A}_{B_s^0 \rightarrow M_1 J/\psi}|^2, \quad (5)$$

can be computed. The  $J/\psi$  mass is noted  $m_{J/\psi}$  while  $m_{M_1} = m_{f_0(980)}$  or  $m_\phi$  denote the  $f_0(980)$  and  $\phi$  masses; the triangle function is  $\lambda(x, y, z) = (x + y - z)^2 - 4xy$ . In Eq. (5),  $\Gamma_{B_s^0} = 1/\tau_{B_s^0}$  is the  $B_s^0$  decay width with  $\tau_{B_s^0} = (1.470 \pm 0.026)$  ps [18] and  $m_{B_s^0}$  is the  $B_s^0$  mass. For the CKM elements in Eqs. (2) and (4) we use the Wolfenstein parametrization,

$$\begin{aligned} \lambda_u &= V_{ub}^* V_{us} = A \lambda^4 (\rho + i\eta), \\ \lambda_c &= V_{cb}^* V_{cs} = A \lambda^2 \left( 1 - \frac{\lambda^2}{2} \right), \end{aligned} \quad (6)$$

with the Wolfenstein parameters  $A = 0.814$ ,  $\rho = 0.1385$ ,  $\eta = 0.358$  and  $\lambda = 0.2257$  [18].

## A. Non-perturbative amplitude

### 1. The case of the scalar-vector decay

The scalar-vector factor,  $A_{f_0 J/\psi}$ , in Eq. (4) is given by,

$$A_{f_0 J/\psi} = \langle f_0(p_{f_0}) | \bar{b} \gamma_\mu (1 - \gamma_5) s | B_s^0(p_{B_s^0}) \rangle \times \langle J/\psi(p_{J/\psi}, \varepsilon_{J/\psi}^*) | \bar{c} \gamma^\mu c | 0 \rangle, \quad (7)$$

where the hadronic matrix element which describes the transition between the  $B_s^0$  and a scalar meson,  $f_0$ , with the respective four-momenta  $p_{B_s^0}$  and  $p_{f_0}$  is [19],

$$\begin{aligned} \langle f_0(p_{f_0}) | \bar{b} \gamma_\mu (1 - \gamma_5) s | B_s^0(p_{B_s^0}) \rangle = \\ \left( p_{B_s^0} + p_{f_0} - \frac{m_{B_s^0}^2 - m_{f_0}^2}{q^2} q \right)_\mu F_1^{B_s^0 \rightarrow f_0}(q^2) \\ + \frac{m_{B_s^0}^2 - m_{f_0}^2}{q^2} q_\mu F_0^{B_s^0 \rightarrow f_0}(q^2), \end{aligned} \quad (8)$$

with  $q = p_{B_s^0} - p_{f_0}$ ,  $q^2 = m_{J/\psi}^2$  and where  $F_1^{B_s^0 \rightarrow f_0}(q^2)$  and  $F_0^{B_s^0 \rightarrow f_0}(q^2)$  are the vector and scalar form factors, respectively. In Eq. (7), the leptonic decay constant,  $f_{J/\psi}$ , of the  $J/\psi$  vector meson, with four-momentum,  $p_{J/\psi}$ , and polarisation,  $\varepsilon_{J/\psi}^*$ , is defined as,

$$\langle J/\psi(p_{J/\psi}, \varepsilon_{J/\psi}^*) | \bar{c} \gamma^\mu c | 0 \rangle = -i f_{J/\psi} m_{J/\psi} \varepsilon_{J/\psi}^{\mu*}. \quad (9)$$

The scalar-vector factor, given by the product of

Eqs. (8) and (9), is then obtained as,

$$A_{f_0 J/\psi} = -i \frac{G_F}{\sqrt{2}} 2m_{J/\psi} \epsilon_{J/\psi}^* \cdot p_{B_s^0} F_1^{B_s^0 \rightarrow f_0}(m_{J/\psi}^2) f_{J/\psi}, \quad (10)$$

with  $4m_{J/\psi}^2 |\epsilon_{J/\psi}^* \cdot p_{B_s^0}|^2 = m_{B_s^0}^2 \lambda^{1/2}(m_{B_s^0}^2, m_{J/\psi}^2, m_{f_0}^2)$  and the Fermi constant,  $G_F = 1.16 \times 10^{-5} \text{GeV}^{-2}$ . The  $B_s^0 \rightarrow f_0$  transition form factor  $F_1^{B_s^0 \rightarrow f_0}(m_{J/\psi}^2)$  will be discussed in Section V.

## 2. The case of the vector-vector decay

For the case of two vector mesons,  $M_1$  and  $M_2$ , the helicity formalism requires the introduction of three polarization four-vectors,  $\epsilon_{M_j, k}$  ( $j = 1, 2$  and  $k = 1, 2, 3$ ) for each spin-1 particle,  $M_j$ ,

$$\begin{aligned} \epsilon_{M_j, 1} &= (0, \vec{\epsilon}_{M_j, 1}), \\ \epsilon_{M_j, 2} &= (0, \vec{\epsilon}_{M_j, 2}), \\ \epsilon_{M_j, 3} &= (|\vec{p}_{M_j}|/m_{M_j}, E_{M_j} \hat{p}_{M_j}/m_{M_j}). \end{aligned} \quad (11)$$

where  $m_{M_j}$ ,  $p_{M_j}$  and  $E_{M_j}$  are the mass, the momentum and the energy of the vector meson,  $M_j$ , respectively. The energies  $E_{M_1}, E_{M_2}$  are given by,

$$E_{M_{1,2}} = \frac{1}{2m_{M_{1,2}}} (m_{B_s^0}^2 - m_{M_1}^2 - m_{M_2}^2). \quad (12)$$

In Eq. (11),  $\hat{p}_{M_j}$  is defined as the unit vector along the momentum:  $\hat{p}_{M_j} = \vec{p}_{M_j}/|\vec{p}_{M_j}|$ .

The three polarization four-vectors,  $\epsilon_{M_j, k}$ , also satisfy the following relations,

$$\epsilon_{M_j, k}^2 = -1, \text{ and } \epsilon_{M_j, k} \cdot \epsilon_{M_j, l} = 0, \text{ for } k \neq l. \quad (13)$$

The vectors  $\vec{\epsilon}_{M_j, 1}$ ,  $\vec{\epsilon}_{M_j, 2}$  and  $\vec{\epsilon}_{M_j, 3}$  form an orthogonal basis in which  $\vec{\epsilon}_{M_j, 1}$  and  $\vec{\epsilon}_{M_j, 2}$  describe the transverse polarizations while  $\vec{\epsilon}_{M_j, 3}$  is the longitudinal polarization vector. With these three vectors one builds up the helicity basis,

$$\begin{aligned} \epsilon_{M_j, +} &= \frac{1}{\sqrt{2}} (\epsilon_{M_j, 1} + i \epsilon_{M_j, 2}) = \frac{1}{\sqrt{2}} (0, +1, i, 0), \\ \epsilon_{M_j, -} &= \frac{1}{\sqrt{2}} (\epsilon_{M_j, 1} - i \epsilon_{M_j, 2}) = \frac{1}{\sqrt{2}} (0, -1, i, 0), \\ \epsilon_{M_j, 0} &= \epsilon_{M_j, 3}. \end{aligned} \quad (14)$$

and  $\epsilon_{M_1, \pm} = \epsilon_{M_2, \mp}$ . In Eq. (14), the new four-vectors  $\epsilon_{M_j, +}$ ,  $\epsilon_{M_j, -}$  and  $\epsilon_{M_j, 0}$  are eigenvectors of the helicity operator corresponding to the eigenvalues  $h = +1, -1$  and 0, respectively.

The vector-vector factor,  $A_{M_1 M_2}^h$ , in Eq. (2) is

$$A_{M_1 M_2}^h = \langle M_1(p_{M_1}, \epsilon_{M_1}^*) | \bar{b} \gamma_\mu (1 - \gamma_5) q | B_s^0(p_{B_s^0}) \rangle \times \langle M_2(p_{M_2}, \epsilon_{M_2}^*) | \bar{q} \gamma^\mu q' | 0 \rangle, \quad (15)$$

where, in the  $B_s^0$  rest-frame, the vector mesons  $M_1$  and  $M_2$  have opposite momentum  $\vec{p}_{M_1} = -\vec{p}_{M_2}$  along the  $z$ -direction and  $\epsilon_{M_j, 0} \cdot p_{M_j} = 0$ .

The matrix hadronic element of a  $P \rightarrow V$  transition can be decomposed into Lorentz invariants as [16, 19, 20]

$$\begin{aligned} \langle M_j(p_{M_j}, \epsilon_{M_j}^*) | \bar{b} \gamma_\mu (1 - \gamma_5) q | B_s^0(p_{B_s^0}) \rangle &= \epsilon_{M_j, \mu}^* (m_{B_s^0} + m_{M_j}) A_1^{B_s^0 \rightarrow M_j}(q^2) - (p_{B_s^0} + p_{M_j})_\mu (\epsilon_{M_j}^* \cdot p_{B_s^0}) \frac{A_2^{B_s^0 \rightarrow M_j}(q^2)}{m_{B_s^0} + m_{M_j}} \\ &\quad - q_\mu (\epsilon_{M_j}^* \cdot p_{B_s^0}) \frac{2m_{M_j}}{q^2} [A_3^{B_s^0 \rightarrow M_j}(q^2) - A_0^{B_s^0 \rightarrow M_j}(q^2)] + i \epsilon_{\mu\nu\alpha\beta} \epsilon_{M_j}^{*\nu} p_{B_s^0}^\alpha p_{M_j}^\beta \frac{2V^{B_s^0 \rightarrow M_j}(q^2)}{m_{B_s^0} + m_{M_j}}, \end{aligned} \quad (16)$$

where the form factors  $A_0^{B_s^0 \rightarrow M_j}(q^2)$ ,  $A_1^{B_s^0 \rightarrow M_j}(q^2)$ ,  $A_2^{B_s^0 \rightarrow M_j}(q^2)$  and  $A_3^{B_s^0 \rightarrow M_j}(q^2)$  obey the following exact relations,

$$\begin{aligned} A_3^{B_s^0 \rightarrow M_j}(q^2) &= \frac{m_{B_s^0} + m_{M_j}}{2m_{M_j}} A_1^{B_s^0 \rightarrow M_j}(q^2) \\ &\quad - \frac{m_{B_s^0} - m_{M_j}}{2m_{M_j}} A_2^{B_s^0 \rightarrow M_j}(q^2), \end{aligned} \quad (17)$$

as well as for  $q^2 = 0$ ,  $A_3^{B_s^0 \rightarrow M_j}(0) = A_0^{B_s^0 \rightarrow M_j}(0)$ .

Specifically for  $M_1 = \phi$ , and  $M_2 = J/\psi$ , the helicity dependent vector-vector factor  $A_{\phi J/\psi}^h$  in Eq. (2) has thus

the following form,

$$\begin{aligned} A_{\phi J/\psi}^{(h=0)} &= i \frac{G_F}{\sqrt{2}} f_{J/\psi} \left[ -m_\phi (m_{B_s^0} + m_\phi) A_1^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2) \right. \\ &\quad \left. + (m_{B_s^0}^2 + m_\phi^2 - m_{J/\psi}^2) A_0^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2) \right]; \end{aligned} \quad (18a)$$

$$A_{\phi J/\psi}^{(h=\pm 1)} = i \frac{G_F}{\sqrt{2}} m_{B_s^0} m_{J/\psi} f_{J/\psi} F_{\mp}^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2). \quad (18b)$$

In Eq. (18b), the transition form factors  $F_{\pm}^{B_s^0 \rightarrow \phi}(q^2) =$

$m_{J/\psi}^2$ ) are

$$F_{\pm}^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2) = \left(1 + \frac{m_{\phi}}{m_{B_s^0}}\right) A_1^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2) \mp \frac{2|\vec{p}_{B_s^0}|}{m_{B_s^0} + m_{\phi}} V^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2), \quad (19)$$

where the center-of-mass momentum  $|\vec{p}_{B_s^0}|$  is defined as,

$$|\vec{p}_{B_s^0}| = \frac{\sqrt{(m_{B_s^0}^2 - M_+^2)(m_{B_s^0}^2 - M_-^2)}}{2m_{B_s^0}}, \quad (20)$$

with  $M_{\pm} = m_{J/\psi} \pm m_{\phi}$ . We note that a somewhat different form for  $A_{\phi J/\psi}^{(h=0)}$  was derived in Ref. [20], which seems to approximate the vector mesons as light mesons. The form factors  $A_0^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2)$  and  $A_1^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2)$  in Eqs. (18a) and (19), as well as  $V^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2)$  in Eq. (19) are defined in Section V. Ref. [14] asserts that when neglecting vector meson masses, Eq. (18a) reduces to,

$$A_{\phi J/\psi}^{(h=0)} = i \frac{G_F}{\sqrt{2}} f_{J/\psi} m_{B_s^0}^2 A_0^{B_s^0 \rightarrow \phi}(m_{J/\psi}^2). \quad (21)$$

The numerical effects in the calculated values of  $B_s^0 \rightarrow J/\psi \phi$  and  $B_s^0 \rightarrow J/\psi f_0(980)$  branching ratios are too important to justify such an approximation.

## B. Perturbative amplitude

The  $a_n^{q,h}(\mu)$  coefficients that appear in Eqs. (2) and (4) are linear combinations of Wilson coefficients,  $C_n(\mu)$ , either at the scale  $\mu = m_b$  or  $m_b/2$  (see below):

$$\begin{aligned} a_n^{q,h}(m_b) &= \left[ C_n(m_b) + \frac{C_{n\pm 1}(m_b)}{N_c} \right] N_n(J/\psi) \\ &+ P_n^{q,h}(J/\psi) + \frac{C_{n\pm 1}(m_b)}{N_c} \frac{C_F}{4\pi} \alpha_s(m_b) V_n^h(J/\psi) \\ &+ \pi C_F \alpha_s(m_b/2) \frac{C_{n\pm 1}(m_b/2)}{N_c^2} H_n^h(M_1 J/\psi). \end{aligned} \quad (22)$$

The superscript,  $(h)$ , explicit the helicity dependence of  $a_n^{q,h}(\mu)$  in the case where  $B_s^0$  decays into two vector mesons. This superscript is dropped in the scalar-vector case. There is no flavor dependence in  $a_n^{q,h}(\mu)$  for  $n = 1, 2$ . In Eq. (22), the upper (lower) signs in  $C_{n\pm 1}(\mu)$  apply when  $n$  is odd (even) and

$$N_n(J/\psi) = 0, \quad n \in \{6, 8\}, \quad \text{else } N_n(J/\psi) = 1. \quad (23)$$

The Wilson coefficients,  $C_n(\mu)$ , in the Naive Dimensional Regularization (NDR) scheme are taken at the hard scale  $m_b$  for the vertex,  $V_n^h(J/\psi)$ , and penguin,  $P_n^{q,h}(J/\psi)$ , corrections, whereas in the hard scattering,  $H_n^h(M_1 J/\psi)$ , amplitudes they are evaluated at  $m_b/2$  since those contributions involve the spectator quark. The strong coupling constants at these scales are  $\alpha_s(m_b) = 0.224$  and

$\alpha_s(m_b/2) = 0.286$  [18], while the number of active flavors is  $n_F = 5$ , the color number  $N_c = 3$  and  $C_F = (N_c^2 - 1)/2N_c$ .

## C. Suppressed higher order corrections and possibility of new physics

There are no contributions, such as given by the annihilation operators derived in Ref. [13], to the two decays considered here. This is because for the final states,  $J/\psi \phi$  and  $J/\psi f_0(980)$ , both mesons are simultaneously flavor and color singlets. At tree level, for instance, the  $W^{\pm}$  exchange diagram produces the charmonium  $\bar{c}c$ , yet the creation of the  $\bar{s}s$  which hadronizes to an  $f_0(980)$  or  $\phi$  must proceed via multiple gluons or by means of photon/ $Z$  exchange. The annihilation is thus either strongly (Zweig) suppressed in  $\alpha_s$  or the suppression is in the electromagnetic coupling constant  $\alpha_{\text{em}}$ .

On the other hand, as will be discussed in Section VI, if we account for vertex, penguin and hard scattering corrections only, the  $B_s^0 \rightarrow J/\psi \phi$  observables are only moderately well reproduced. As can be seen in Table IX, the branching ratio, for instance, is about 20% too large (although still within the experimental errors). We therefore allow for additional phenomenological amplitudes that mock up “other” contributions, be it from annihilation topologies expected to be strongly suppressed or possible physics beyond the Standard Model [21]. These are included in Eqs. (2) and (4) with the amplitudes,  $\zeta^h$  and  $\zeta$ , conveniently scaled as,

$$\zeta^{(h)} = \frac{B_{M_1 J/\psi}}{A_{M_1 J/\psi}^{(h)}} X_C. \quad (24)$$

The factor  $B_{M_1 J/\psi}$  is chosen to be a product of decay constants, either

$$B_{f_0 J/\psi} = -i \frac{G_F}{\sqrt{2}} f_{B_s^0} \bar{f}_{f_0} f_{J/\psi}, \quad (25)$$

if  $M_1 = f_0(980)$  or

$$B_{\phi J/\psi} = i \frac{G_F}{\sqrt{2}} f_{B_s^0} f_{\phi} f_{J/\psi}, \quad (26)$$

if  $M_1 = \phi$ , while the factor  $X_C$  is a complex parameter discussed in Section V C. We note that the decay constant,  $f_{f_0}$ , vanishes due to charge conjugation invariance, wherefore the scalar light cone distributions amplitude (LCDA) is normalized to  $\bar{f}_{f_0} = f_{f_0} m_{f_0} / (m_{u,d}(\mu) - m_{u,d}(\mu))$ , which is finite [22]. We shall return to this issue in Section IV.

## D. The ratio $\mathcal{R}_{f_0/\phi}$

Prior to discussing the various  $\alpha_s(\mu)$  corrections to the amplitudes,  $a_n^{p,h}(\mu)$ , it may be of interest to observe the

qualitative behavior of the ratio,  $\mathcal{R}_{f_0/\phi}$ , in terms of the scales  $\Lambda_{\text{QCD}}$  and  $m_b$ . A naive factorization analysis yields a hierarchy of helicity amplitudes for  $B$  into vector-vector decays [14],

$$\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=0)} : \mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=+1)} : \mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=-1)} \\ \Longleftrightarrow 1 : \frac{\Lambda_{\text{QCD}}}{m_b} : \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^2, \quad (27)$$

while for  $\bar{B}_s$  mesons the signs are exchanged ( $h = +1 \rightarrow h = -1$ ). Furthermore, the amplitudes  $\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=0)}$  and  $\mathcal{A}_{B_s^0 \rightarrow f_0 J/\psi}$  are of same order in  $\Lambda_{\text{QCD}}/m_b$ . With this estimation, the ratio  $\mathcal{R}_{f_0/\phi}$  we are interested in becomes,

$$\mathcal{R}_{f_0/\phi} = \frac{|\mathcal{A}_{B_s^0 \rightarrow f_0 J/\psi}|^2}{|\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=0)}|^2 + |\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=-1)}|^2 + |\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=+1)}|^2} \\ \simeq \mathcal{O}(1) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^4. \quad (28)$$

Hence,  $\mathcal{R}_{f_0/\phi}$  is  $\mathcal{O}(1)$  for  $\Lambda_{\text{QCD}}/m_b$  corrections.

Nonetheless, non-perturbative hadronic effects can spoil the naive factorization and violate the hierarchy in Eq. (27); so do electromagnetic penguin contributions where a photon with small virtuality subsequently converts into a vector meson [23].

### III. QCDF CORRECTIONS FOR $B_s^0 \rightarrow \phi J/\psi$ DECAY AMPLITUDES

Due to the structure of the four-quark operators in heavy quark effective theory and the conservation of the flavor quantum numbers, the final state  $M_1 M_2 = \phi J/\psi$  is created from the transition  $B_s^0 \rightarrow \phi$  and the production of  $J/\psi$  from vacuum. As discussed in Section II, the decay amplitudes at leading order in  $\Lambda_{\text{QCD}}/m_b$  and  $\alpha_s(m_b)$  are given by the factorized product of a transition form factor and a decay constant. Following Ref. [14], we only give QCD corrections that explicitly appear in the amplitude  $\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^h$  of Eq. (2).

We discard terms proportional to  $r = (m_{J/\psi}/m_{B_s})^2 \simeq 1/3$  in vertex corrections which stem from the presence of the charm quark in the loop diagram; we have numerically checked that their contributions to the  $a_n^{q,h}(\mu)$  coefficients are negligible, all the more so when seen in the light of the large hadronic uncertainties of the form factors [see Sections (VA) and (VB)]. We note that in the limit  $r \rightarrow 0$ , one recovers the vertex correction known from, for example,  $B \rightarrow \pi\pi$  which is of course infrared safe.

Since the coefficients in the Gegenbauer expansion of the LCDA are poorly known for the scalar mesons, and only with non-negligible errors for the vector mesons  $V = \phi$  and  $V = J/\psi$ , we limit ourselves to leading terms in the expansion. The leading twist-2 distribution and twist-3

two particle distribution amplitudes are approximated by

$$\phi_V(x) = 6x(1-x) \quad (29)$$

and

$$\varphi_V(x) = 3(2x-1), \quad (30)$$

respectively. In the annihilation and hard scattering amplitudes, the chiral coefficient,  $r_\chi^V(\mu)$ , is defined as

$$r_\chi^V(\mu) = \frac{2m_V}{m_b(\mu)} \frac{f_V^\perp(\mu)}{f_V} \simeq \frac{2m_V}{m_b(\mu)}, \quad (31)$$

where  $f_V^\perp(\mu)$  is the transverse decay constant for any vector  $V$  and  $\mu = m_b/2$ .

#### A. Penguin contributions

The penguin contributions to the amplitude in Eq. (2) stems from the positive helicity,  $h = +1$ , amplitudes  $P_{7,9}^{q,h=+1}(J/\psi)$  given in Ref. [14],

$$P_{7,9}^{q,h=+1}(J/\psi) = -\frac{\alpha_e}{3\pi} C_{7\gamma}^{\text{eff}}(\mu) \frac{m_{B_s} m_b}{m_{J/\psi}^2} + \frac{2\alpha_e}{27\pi} \\ \times \left( C_1(\mu) + N_c C_2(\mu) \right) \left[ \delta_{qc} \ln \frac{m_c^2}{\mu^2} + \delta_{qu} \ln \frac{\nu^2}{\mu^2} + 1 \right], \quad (32)$$

whereas  $P_{7,9}^{q,h=-1}(J/\psi) = 0$ . In Eq. (32),  $\mu = m_b$ ,  $C_{7\gamma}^{\text{eff}}(\mu) = C_{7\gamma}(\mu) - C_5(\mu)/3 - C_6(\mu)$ ,  $\alpha_e = 1/129$  is the electromagnetic coupling constant and the scale  $\nu$  refers to the  $f_{J/\psi}$  decay constant scale. One also has  $P_{3,5}^{q,h=\pm 1}(J/\psi) = 0$  as well as  $P_{3,5,7,9}^{q,(h=0)}(J/\psi) = 0$ .

#### B. Vertex contributions

In  $B_s^0 \rightarrow \phi J/\psi$ , the electroweak vertex receives  $\alpha_s(\mu)$  corrections to all  $a_n^{q,h}(\mu)$  in the amplitudes  $\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^h$ . For  $h = 0$ , these are, with  $\mu = m_b$ ,

$$V_n^{h=0}(J/\psi) = \begin{cases} 12 \ln \left( \frac{m_b}{\mu} \right) - 3i\pi - \frac{27}{2}, & \text{for } n \in \{2, 3, 9\} \\ -12 \ln \left( \frac{m_b}{\mu} \right) + 3i\pi + \frac{13}{2}, & \text{for } n \in \{5, 7\} \end{cases} \quad (33)$$

whereas for  $h = -1$  one has,

$$V_n^{h=-1}(J/\psi) = \begin{cases} 12 \ln\left(\frac{m_b}{\mu}\right) + \pi^2 - \frac{143}{4}, & \text{for } n \in \{2, 3, 9\} \\ -12 \ln\left(\frac{m_b}{\mu}\right) - \pi^2 + \frac{95}{4}, & \text{for } n \in \{5, 7\} \end{cases} \quad (34)$$

and for  $h = +1$  one has,

$$V_n^{h=+1}(J/\psi) = \begin{cases} 12 \ln\left(\frac{m_b}{\mu}\right) + \frac{\pi^2}{2} - 6i\pi - \frac{71}{4}, & \text{for } n \in \{2, 3, 9\} \\ -12 \ln\left(\frac{m_b}{\mu}\right) - \frac{\pi^2}{2} + 6i\pi + \frac{23}{4}, & \text{for } n \in \{5, 7\}. \end{cases} \quad (35)$$

### C. Hard scattering contributions

The gluon exchange between a  $J/\psi$  meson and the spectator  $s$ -quark leads to the hard scattering amplitudes,

$$H_n^{h=0}(\phi J/\psi) = \pm 3 \frac{B_{\phi J/\psi}}{A_{\phi J/\psi}^{h=0}} \frac{m_{B_s^0}}{\lambda_{B_s^0}} \left( r_\chi^\phi(\mu) X_H + 3 \right), \quad (36)$$

for  $h = 0$ ,  $\mu = m_b/2$  and  $\lambda_{B_s^0} = 0.350$  GeV [13]. The plus sign is for  $n = 2, 3, 9$  and the minus sign for  $n = 5, 7$ . The phenomenological amplitude,  $X_H$ , parametrizes the end-point divergence of the scalar meson's LCDA and is defined in Eq. (54). For the helicity,  $h = +1$ , the correction reads

$$H_n^{h=+1}(\phi J/\psi) = \mp 18 \frac{B_{\phi J/\psi}}{A_{\phi J/\psi}^{h=+1}} \frac{f_\phi^\perp}{f_\phi} \frac{m_{J/\psi}}{\lambda_{B_s^0}} (X_H - 1), \quad (37)$$

where the minus sign applies to  $n = 2, 3, 9$  and the plus sign to  $n = 5, 7$ . The helicity,  $h = -1$ , contribution is simply,

$$H_n^{h=-1}(\phi J/\psi) = 0 \text{ for } n = 2, 3, 5, 7, 9. \quad (38)$$

### IV. QCDF CORRECTIONS FOR $B_s^0 \rightarrow f_0(980)J/\psi$ DECAY AMPLITUDES

We now turn to the  $B_s^0 \rightarrow J/\psi f_0(980)$  transition for which the  $\alpha_s(\mu)$  corrections are all included following Ref. [13] applied to an  $SV$  final state. For previously mentioned reasons, we solely employ the first non-vanishing leading term in the LCDA,

$$\phi_{f_0}(x) = 6x(1-x) \left[ 3B_1(\mu)(2x-1) \right], \quad (39)$$

where  $B_1(m_b/2) = -0.54$  [22] is the  $f_0(980)$ 's first Gegenbauer moment and we remind that only odd moments contribute in case of charge-neutral scalar mesons. In particular, contrary to the pseudoscalar LCDA, the leading term  $6x(1-x)B_0$  vanishes since  $B_0 = (m_1(\mu) - m_2(\mu))/m_S$ , where  $m_S$  is the scalar meson mass and  $m_{1,2}(\mu)$  its running quark masses. The scalar twist-3 two-particle distribution is given by

$$\varphi_{f_0}(x) = 1. \quad (40)$$

The asymptotic forms of the LCDA,  $\phi_{J/\psi}(x)$  (Eq. (29)) and  $\varphi_{J/\psi}(x)$  (Eq. (30)), are used. As in the  $B_s^0 \rightarrow \phi J/\psi$  decay, the  $J/\psi$  meson is created from vacuum whereas the transition  $B_s^0 \rightarrow f_0(980)$  produces the scalar meson. Here, we only consider the  $s\bar{s}$  component of the  $f_0(980)$  since the flavor of the spectator quark in the tree and penguin topologies of  $B_s^0$  decays is strange. There are no penguin corrections [13] to the  $B_s^0 \rightarrow f_0(980)J/\psi$  decay amplitude in Eq. (4).

### A. Vertex contributions

At the order of  $\alpha_s(\mu)$ , the vertex correction,  $V_n(J/\psi)$ , involves the leading twist distribution,  $\phi_{J/\psi}(x)$ , and a gluon kernel given in [13]. We derive from this the expressions,

$$V_n(J/\psi) = \begin{cases} 12 \ln\left(\frac{m_b}{\mu}\right) - 3i\pi - \frac{37}{2}, & \text{for } n \in \{2, 3, 9\} \\ -12 \ln\left(\frac{m_b}{\mu}\right) + 3i\pi + \frac{13}{2}, & \text{for } n \in \{5, 7\} \end{cases} \quad (41)$$

with  $\mu = m_b$ .

### B. Hard scattering contributions

The hard scattering correction in case of an  $f_0 J/\psi$  final state reads

$$H_n(f_0 J/\psi) = \pm 3 \frac{B_{f_0 J/\psi}}{A_{f_0 J/\psi}} \frac{m_{B_s^0}}{\lambda_{B_s^0}} \left( \bar{r}_\chi^{f_0}(\mu) X_H + 3B_1(\mu) \right), \quad (42)$$

where the plus sign applies to  $n = 2, 3, 9$ , the minus sign to  $n = 5, 7$  and  $X_H$  is given, as in the case of the  $\phi J/\psi$  final state, by Eq. (54).

The chiral coefficient,  $\bar{r}_\chi^{f_0}(\mu)$ , enters Eq. (42) rather than  $r_\chi^{f_0}(\mu)$  defined as,

$$r_\chi^{f_0}(\mu) = \frac{2m_{f_0}^2}{m_b(\mu)(m_1(\mu) - m_2(\mu))}. \quad (43)$$

TABLE I: Wilson coefficients at the  $\mu = m_b$  and  $\mu = m_b/2$  scales in the NDR scheme [12]. The coefficients  $C_7(\mu) - C_{10}(\mu)$  must be multiplied by  $\alpha_e$ .

	$C_1(\mu)$	$C_2(\mu)$	$C_3(\mu)$	$C_4(\mu)$	$C_5(\mu)$	$C_6(\mu)$	$C_7(\mu)$	$C_8(\mu)$	$C_9(\mu)$	$C_{10}(\mu)$	$C_{7\gamma}(\mu)$
$\mu = m_b$	1.081	-0.190	0.014	-0.036	0.009	-0.042	-0.011	0.06	-1.254	0.233	-0.318
$\mu = m_b/2$	1.137	-0.295	0.021	-0.051	0.010	-0.065	-0.24	0.096	-1.325	0.331	-0.364

TABLE II: Values of the higher order correction ( $\rho_C$ ,  $\phi_C$ ) and hard-scattering ( $\rho_H$ ,  $\phi_H$ ) parameters as function of the  $B_s^0$  decay constant.

$f_{B_s^0}$ [MeV]	$\rho_C$	$\phi_C$ (°)	$\rho_H$	$\phi_H$ (°)
230	$4.52 \pm 2.24$	$173.8 \pm 37.6$	$1.90 \pm 0.20$	$266.0 \pm 21.6$
260	$6.16 \pm 2.03$	$176.1 \pm 53.6$	$1.70 \pm 0.16$	$260.6 \pm 19.3$
290	$7.33 \pm 1.63$	$176.0 \pm 57.6$	$1.54 \pm 0.15$	$255.6 \pm 17.3$

The reason is that in case of neutral scalar mesons,  $m_1(\mu) = m_2(\mu)$  and  $r_\chi^{f_0}(\mu)$  diverges. On the other hand, it is known from  $C$ -conjugation invariance that the vector decay constant of the neutral scalar meson must vanish. However, the quark equations of motions yield a relation between the scalar- and vector-decay constants,  $\bar{f}_{f_0}$  and  $f_{f_0}$  respectively:

$$\bar{f}_{f_0} = \frac{m_{f_0}}{m_1(\mu) - m_2(\mu)} f_{f_0}, \quad (44)$$

where  $m_{f_0} \bar{f}_{f_0} = \langle 0 | \bar{q}_2 q_1 | f_0 \rangle$ . Since  $\bar{f}_{f_0}$  is non-zero, the product  $f_{f_0} m_{f_0} / (m_1(\mu) - m_2(\mu))$  is finite in the limit  $m_1(\mu) \rightarrow m_2(\mu)$ . We thus recombine,  $f_{f_0} r_\chi^{f_0} = \bar{f}_{f_0} \bar{r}_\chi^{f_0}$ , with

$$\bar{r}_\chi^{f_0}(\mu) = \frac{2m_{f_0}}{m_b(\mu)}. \quad (45)$$

## V. NUMERICAL PARAMETERS

This section serves to summarize all parameter values required for numerical applications. The Wilson coefficients at the scales  $\mu = m_b$  and  $\mu = m_b/2$  used in this work are listed in Table I. For the meson masses, we refer to the latest PDG values [18], which are (in GeV):

$$m_{B_s^0} = 5.366, \quad m_{B_s^*} = 5.412, \quad m_{f_0} = 0.980, \\ m_{J/\psi} = 3.096, \quad m_\phi = 1.019. \quad (46)$$

The running quark masses at  $\mu = m_b = 4.2$  GeV are (in GeV),

$$m_b = 4.2, \quad m_c = 1.3, \quad m_s = 0.07, \quad m_{u,d} = 0.003, \quad (47)$$

and those at  $\mu = m_b/2 = 2.1$  GeV are,

$$m_b = 4.95, \quad m_c = 1.51, \quad m_s = 0.09, \quad m_{u,d} = 0.005. \quad (48)$$

We take the  $\phi$  decay constant values from Ref. [14]:  $f_\phi = (221 \pm 3)$  MeV and  $f_\phi^\perp = (175 \pm 25)$  MeV. For the  $J/\psi$  meson, we use  $f_{J/\psi} = (416 \pm 6)$  MeV [24] and  $f_{J/\psi}^\perp = (405 \pm 5)$  MeV [16]. In the  $B_s \rightarrow J/\psi f_0(980)$  channel, the  $s\bar{s}$  component of the  $f_0(980)$  is involved which implies the poorly known scalar decay constant  $\bar{f}_{f_0}$ : one theoretical estimate yields  $\bar{f}_{f_0^s} = (180 \pm 15)$  MeV [25] whereas a much larger value  $\bar{f}_{f_0^s}(1 \text{ GeV}) = (370 \pm 20)$  MeV [ $\bar{f}_{f_0^s}(2.1 \text{ GeV}) = (460 \pm 25)$  MeV] is found in Ref. [22], both from coupling to the scalar  $s\bar{s}$  current only (denoted by the superscript  $s$  in  $f_0^s$ , which we use henceforth). Similarly, several theoretical predictions exist for the leptonic  $B_s$  decay constants of which we select three values from unquenched lattice QCD:  $f_{B_s^0} = (204 \pm 12_{-23}^{+24})$  MeV [26],  $f_{B_s^0} = (259 \pm 32)$  MeV [27] and  $f_{B_s^0} = (231 \pm 15)$  MeV [28].

To illustrate the sensitivity of the ratio  $\mathcal{R}_{f_0/\phi}$  to the hadronic uncertainties, we exemplarily choose three different values for each decay constant:  $f_{B_s^0} = 230, 260, 290$  MeV and  $\bar{f}_{f_0^s} = 340, 380, 420$  MeV.

### A. $B \rightarrow V$ transition form factor

Values for the  $B_s^0 \rightarrow \phi$  transition form factors are taken from the pole-extrapolation model by Melikhov [19]:

$$A_0(q^2)^{B_s^0 \rightarrow \phi} = \frac{a_0(0)}{\left(1 - \frac{q^2}{m_{B_s^0}^2}\right) \left(1 - \sigma_1 \frac{q^2}{m_{B_s^0}^2} + \sigma_2 \frac{q^4}{m_{B_s^0}^4}\right)}. \quad (49)$$

The form factor  $V(q^2)^{B_s^0 \rightarrow \phi}$  is given by a similar expression in which  $a_0(0)$  is replaced by  $v(0)$  and  $m_{B_s^0}$  by  $m_{B_s^*}$  [19]. Next, the  $A_1(q^2)^{B_s^0 \rightarrow \phi}$  form factor is parametrized by

$$A_1(q^2)^{B_s^0 \rightarrow \phi} = \frac{a_1(0)}{\left(1 - \sigma_1 \frac{q^2}{m_{B_s^*}^2} + \sigma_2 \frac{q^4}{m_{B_s^*}^4}\right)}. \quad (50)$$

Finally,  $A_2(q^2)^{B_s^0 \rightarrow \phi}$  has the same functional form as  $A_1(q^2)^{B_s^0 \rightarrow \phi}$  where  $a_1(0)$  is replaced by  $a_2(0)$ . In both, Eqs. (49) and (50), the momentum transfer is  $q^2 = m_{J/\psi}^2$ .

TABLE III: Short-distance amplitudes,  $a_n^{q,h}(m_b) \times 10^3$ , for the helicity  $h = +1$  in  $B_s^0 \rightarrow J/\psi\phi$ , as a function of the decay constant,  $f_{B_s^0}$ , and with  $\bar{f}_{f_0^s} = 380$  MeV. The *LOVP* results are obtained with the leading order (*LO*) amplitude to which vertex *V* and penguin *P* corrections are added. In case of *LOVPH*, the hard scattering contribution with the endpoint parametrization  $X_H$  is included. *LOVPH + C* contains additionally the purely phenomenological contribution  $\zeta^{(h)}$  with two more parameters.

$f_{B_s^0}$ [MeV]	<i>LOVP</i>	230		260		290	
		<i>LOVPH</i>	<i>LOVPH + C</i>	<i>LOVPH</i>	<i>LOVPH + C</i>	<i>LOVPH</i>	<i>LOVPH + C</i>
$a_2^{u,c}(m_b)$	$60.38 - i 161.7$	$-3.77 + i 148.8$	$-8.43 + i 129.87$	$38.06 + i 149.7$	$8.21 + i 130.04$	$75.52 + i 140.4$	$22.57 + i 128.49$
$a_3^{u,c}(m_b)$	$5.66 + i 5.39$	$8.54 - i 8.54$	$8.75 - i 7.69$	$6.66 - i 8.58$	$8.0 - i 7.70$	$4.98 - i 8.17$	$7.36 - i 7.63$
$a_5^{u,c}(m_b)$	$-5.27 - i 6.28$	$-8.94 + i 11.47$	$-9.21 + i 10.39$	$-6.55 + i 11.52$	$-8.25 + i 10.40$	$-4.41 + i 10.99$	$-7.43 + i 10.31$
$a_7^u(m_b)$	$0.12 + i 0.07$	$0.17 - i 0.13$	$0.17 - i 0.12$	$0.14 - i 0.13$	$0.16 - i 0.12$	$0.11 - i 0.13$	$0.15 - i 0.12$
$a_7^c(m_b)$	$0.69 + i 0.07$	$0.73 - i 0.13$	$0.74 - i 0.12$	$0.71 - i 0.13$	$0.73 - i 0.12$	$0.68 - i 0.13$	$0.72 - i 0.12$
$a_9^u(m_b)$	$-9.25 - i 0.27$	$-9.40 + i 0.43$	$-9.41 + i 0.39$	$-9.30 + i 0.43$	$-9.37 + i 0.39$	$-9.22 + i 0.41$	$-9.34 + i 0.38$
$a_9^c(m_b)$	$-8.68 - i 0.27$	$-8.83 + i 0.43$	$-8.84 + i 0.39$	$-8.73 + i 0.43$	$-8.80 + i 0.39$	$-8.65 + i 0.41$	$-8.77 + i 0.38$

TABLE IV: Short-distance amplitudes,  $a_n^{q,h}(m_b) \times 10^3$ , for helicity  $h = -1$  and  $B_s^0 \rightarrow J/\psi\phi$ . Since the hard-scattering contributions are zero, these amplitudes are independent of  $f_{B_s^0}$ .

$a_2^{u,c}(m_b)$	-51.72
$a_3^{u,c}(m_b)$	9.39
$a_5^{u,c}(m_b)$	-9.63
$a_7^{u,c}(m_b)$	0.12
$a_9^{u,c}(m_b)$	-9.49

In Eqs. (49) and (50), the form factors at  $q^2 = 0$  are  $a_0(0) = 0.42$  ( $v(0) = 0.44$ ) and  $a_1(0) = 0.34$  ( $a_2(0) = 0.31$ ). The extrapolation parameters are, for  $A_0(q^2)^{B_s^0 \rightarrow \phi}$ ,  $\sigma_1 = 0.55$  and  $\sigma_2 = 0.12$ ; for  $V(q^2)^{B_s^0 \rightarrow \phi}$ ,  $\sigma_1 = 0.62$  and  $\sigma_2 = 0.20$ ; for  $A_1(q^2)^{B_s^0 \rightarrow \phi}$ ,  $\sigma_1 = 0.73$  and  $\sigma_2 = 0.42$  and finally for  $A_2(q^2)^{B_s^0 \rightarrow \phi}$ ,  $\sigma_1 = 1.30$  and  $\sigma_2 = 0.52$ . The respective values for the form factors at the value  $q^2 = m_{J/\psi}^2$  are  $A_0(q^2)^{B_s^0 \rightarrow \phi} = 0.76$ ,  $A_1(q^2)^{B_s^0 \rightarrow \phi} = 0.42$ ,  $A_2(q^2)^{B_s^0 \rightarrow \phi} = 0.49$  and  $V(q^2)^{B_s^0 \rightarrow \phi} = 0.80$ .

### B. $B \rightarrow S$ transition form factor

We studied the transition form factor,  $F_{0,1}^{B_s^0 \rightarrow f_0^s}(q^2)$ , in a comparative calculation using a dispersion relation and a covariant light front dynamics model [10]. To our knowledge, this form factor has only been calculated recently in QCD sum rules [29, 30] and pQCD [31] for  $q^2 =$

0 and must be extrapolated to the value  $F_{0,1}^{B_s^0 \rightarrow f_0^s}(m_{J/\psi}^2)$ .

In our work [10], the transition form factors are derived from the constituent quark three-point function, the vertices of which are the weak interaction coupling,  $\gamma_\mu(1-\gamma_5)$ , and two phenomenological Bethe-Salpeter amplitudes for the  $B_{(s)}$  and  $f_0(980)$  mesons. While the  $B_s$  can be parametrized with the leptonic decay constant (known from lattice-QCD simulations), the latter is more problematic since the  $\bar{f}_{f_0^s}$  is poorly determined. In an attempt to formulate a suitable scalar  $f_0(980)$  vertex function, we constrained its parameters by means of experimental quasi two-body branching fractions,  $D_{(s)} \rightarrow f_0(980)P$ ,  $P = \pi, K$ . The advantage is that the  $F_+^{B_s^0 \rightarrow f_0^s}(q^2)$  and  $F_-^{B_s^0 \rightarrow f_0^s}(q^2)$  form factors,

$$\begin{aligned} \langle f_0^s(p_2) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B_s^0(p_1) \rangle = \\ F_+^{B_s^0 \rightarrow f_0^s}(q^2)(p_1 + p_2)_\mu + F_-^{B_s^0 \rightarrow f_0^s}(q^2)(p_1 - p_2)_\mu, \end{aligned} \quad (51)$$

can be calculated for any physical time-like momentum



TABLE V: As in Table III but for the helicity  $h = 0$ .

$f_{B_s^0}$ [MeV]	$LOVP$	230		260		290	
		$LOVPH$	$LOVPH + C$	$LOVPH$	$LOVPH + C$	$LOVPH$	$LOVPH + C$
$a_2^{u,c}(m_b)$	$54.51 - i 80.86$	$160.2 - i 132.6$	$161.0 - i 129.4$	$165.7 - i 132.7$	$170.6 - i 129.5$	$171.8 - i 131.2$	$180.6 - i 129.2$
$a_3^{u,c}(m_b)$	$5.86 + i 2.69$	$1.11 + i 5.01$	$1.08 + i 4.87$	$0.87 + i 5.02$	$0.65 + i 4.87$	$0.60 + i 4.95$	$0.20 + i 4.86$
$a_5^{u,c}(m_b)$	$-7.17 - i 3.14$	$-1.12 - i 6.10$	$-1.08 - i 5.92$	$-0.81 - i 6.11$	$-0.53 - i 5.92$	$-0.46 - i 6.02$	$0.04 - i 5.90$
$a_7^{u,c}(m_b)$	$0.09 + i 0.03$	$0.02 + i 0.07$	$0.02 + i 0.07$	$0.02 + i 0.07$	$0.02 + i 0.07$	$0.02 + i 0.07$	$0.01 + i 0.06$
$a_9^{u,c}(m_b)$	$-9.31 - i 0.14$	$-9.07 - i 0.25$	$-9.07 - i 0.24$	$-9.06 - i 0.25$	$-9.05 - i 0.24$	$-9.05 - i 0.25$	$-9.03 - i 0.24$

transfer  $q^2 = (p_1 - p_2)^2$ . The superscript  $s$  is a reminder that the transition is to the  $\bar{s}s$  component of the scalar meson and  $p_1$  and  $p_2$  are the  $B_s^0$  and  $f_0(980)$  four-momenta, respectively. We do stress that the  $B_s \rightarrow f_0(980)$  form factor calculated by us in Ref. [10] does not assume a pure  $\bar{s}s$  state of the  $f_0(980)$ . Instead, it was treated as a mixture of strange and non-strange  $\bar{q}q$  components related by a mixing angle which also yields the related form factor  $F_{0,1}^{B \rightarrow f_0^{u,d}}(q^2)$ . This angle was determined with experimental constraints [10] and the overall normalization of the transition form factor receives contributions from both states.

The form factors  $F_{\pm}(q^2)$  (we suppress the flavor superscripts) are related to the set of vector and scalar form factors as,

$$F_1(q^2) = F_+(q^2), \quad (52)$$

$$F_0(q^2) = F_+(q^2) + \frac{q^2}{m_{B_s^0}^2 - m_{f_0}^2} F_-(q^2). \quad (53)$$

The form factor  $F_1(q^2)$  we obtain in both the dispersion relation and covariant light front dynamics approaches agree at the maximum recoil point  $q^2 = 0$ . At large four-momentum transfer, specifically for  $q^2 = m_{J/\psi}^2 \simeq 10 \text{ GeV}^2$ , our model predictions differ significantly which is also known to occur for  $B \rightarrow \pi$  transition form factors [32]. This is not surprising, as for large momentum transfers the final-state meson is less energetic and the soft physics of the bound states becomes more relevant. Since the models differ in their parametrization of the bound-state wave functions, it is clear that their inaccuracies are revealed in the form-factor predictions at large  $q^2$ . In Ref. [30], we deduce from the author's extrapolation parametrization that  $F_1^{B_s^0 \rightarrow f_0^s}(m_{J/\psi}^2) \simeq 0.3$ , which is compatible with our dispersion-relation prediction  $\simeq 0.4$  within the errors. In Section VI, we will account for this rather large window of values and plot the ratio  $\mathcal{R}_{f_0/\phi}$  as a function of  $F_1^{B_s^0 \rightarrow f_0^s}(m_{J/\psi}^2)$ .

### C. Model parameters

The hard scattering contributions involve endpoint divergences, which we choose to parametrize by,

$$X_H = \left(1 + \rho_H \exp(i\phi_H)\right) \ln \frac{m_{B_s^0}}{\lambda_h}. \quad (54)$$

In case of a possible annihilation or “other” contribution we simply write,

$$X_C = \rho_C \exp(i\phi_C) \quad (55)$$

which introduces four parameters,  $0 < \rho_{C,H}$  and  $0 < \phi_{C,H} < 360^\circ$ . We assume that  $X_{C,H}^{h=0} = X_{C,H}^{h=-1} = X_{C,H}^{h=+1} = X_{C,H}$ , as the vector  $\phi$  and scalar  $f_0(980)$  mesons have similar masses and we consider the  $s\bar{s}$  component only. The hard scattering corrections are expected to be of the order of  $m_{B_s^0}/\lambda_h$  in Eq. (54), with  $\lambda_h = 0.5 \text{ GeV}$ . The parameters  $\rho_{C,H}$  and  $\phi_{C,H}$  are chosen so as to reproduce the experimental data discussed in Section VI. We insert their values in the  $B_s^0 \rightarrow J/\psi f_0$  decay amplitude (4) and then predict the branching ratio  $\mathcal{B}(B_s^0 \rightarrow f_0 J/\psi)$ .

## VI. RESULTS AND EXPERIMENTAL DATA

In the  $B_s^0 \rightarrow \phi J/\psi$  decay, one can define five observables: a longitudinal, parallel and perpendicular polarization fraction,  $f_L, f_{\parallel}$  and  $f_{\perp}$ , respectively,

$$f_k = \frac{|\mathcal{A}_k|^2}{|\mathcal{A}_L|^2 + |\mathcal{A}_{\parallel}|^2 + |\mathcal{A}_{\perp}|^2}, \quad k = L, \parallel, \perp \quad (56)$$

as well as two relative phases,  $\phi_{\parallel}$  and  $\phi_{\perp}$ ,

$$\phi_k = \arg\left(\frac{\mathcal{A}_k}{\mathcal{A}_L}\right), \quad k = \parallel, \perp, \quad (57)$$

where we have abbreviated,  $\mathcal{A}_L = \mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=0)}$ ,  $\mathcal{A}_{\parallel} = [\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=+1)} + \mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=-1)}]/\sqrt{2}$  and  $\mathcal{A}_{\perp} = [\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=+1)} -$

TABLE VI: Short-distance amplitudes,  $a_n^q(m_b) \times 10^3$ , for  $B_s^0 \rightarrow J/\psi f_0(980)$  as a function of the  $f_{B_s^0}$  decay constant with  $\bar{f}_{f_0^s} = 380$  MeV and  $F_1^{B_s^0 \rightarrow f_0^s}(m_{J/\psi}^2) = 0.4$ . See caption in Table III for the definition of *LOVP*, *LOVPH* and *LOVPH + C* amplitudes.

$f_{B_s^0}$ [MeV]	<i>LOVP</i>	230		260		290	
		<i>LOVPH</i>	<i>LOVPH + C</i>	<i>LOVPH</i>	<i>LOVPH + C</i>	<i>LOVPH</i>	<i>LOVPH + C</i>
$a_2^{u,c}(m_b)$	$11.61 - i 80.86$	$-42.40 - i 255.5$	$-33.35 - i 224.3$	$-66.51 - i 234.1$	$-51.82 - i 224.4$	$-95.23 - i 229.5$	$-69.17 - i 223.8$
$a_3^{u,c}(m_b)$	$7.29 + i 2.69$	$9.71 + i 10.53$	$9.30 + i 9.13$	$10.80 + i 9.57$	$10.13 + i 9.13$	$12.08 + i 9.36$	$10.91 + i 9.10$
$a_5^{u,c}(m_b)$	$-7.17 - i 3.14$	$-10.25 - i 13.12$	$-9.74 - i 11.34$	$-11.63 - i 11.90$	$-10.79 - i 11.35$	$-13.27 - i 11.64$	$-11.78 - i 11.31$
$a_7^{u,c}(m_b)$	$0.09 + 0.03$	$0.13 + i 0.15$	$0.12 + i 0.13$	$0.14 + i 0.14$	$0.14 + i 0.13$	$0.16 + i 0.13$	$0.15 + i 0.13$
$a_9^{u,c}(m_b)$	$-9.38 - i 0.14$	$-9.51 - i 0.53$	$-9.49 - i 0.46$	$-9.56 - i 0.48$	$-9.53 - i 0.46$	$-9.63 - i 0.47$	$-9.57 - i 0.46$

TABLE VII: The phenomenological contributions  $\zeta^h \times 10^3$  for  $h = 0, -1, +1$ , Eq. (24), to the  $B_s^0 \rightarrow J/\psi \phi$  amplitude as a function of the  $f_{B_s^0}$  decay constant with  $\bar{f}_{f_0^s} = 380$  MeV.

$f_{B_s^0}$ [MeV]	230	260	290
$\zeta^{h=0}$	$-18.11 + i 1.98$	$-28.04 + i 1.89$	$-37.19 + i 2.63$
$\zeta^{h=-1}$	$-129.26 + i 14.12$	$-200.12 + i 13.46$	$-265.41 + i 18.77$
$\zeta^{h=+1}$	$-15.25 + i 1.67$	$-23.61 + i 1.59$	$-31.31 + i 2.21$

$$\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=-1)}] / \sqrt{2}.$$

The *CP* average is defined in terms of the polarization fractions,  $f_k$ ,

$$A_{\text{CP}}^k = \frac{f_k^{\bar{B}_s^0} - f_k^{B_s^0}}{f_k^{\bar{B}_s^0} + f_k^{B_s^0}}. \quad (58)$$

Similarly, for  $B_s^0 \rightarrow f_0(980)J/\psi$ , the *CP* average is defined as,

$$A_{\text{CP}} = \frac{\mathcal{B}(\bar{B}_s^0 \rightarrow f_0 J/\psi) - \mathcal{B}(B_s^0 \rightarrow f_0 J/\psi)}{\mathcal{B}(\bar{B}_s^0 \rightarrow f_0 J/\psi) + \mathcal{B}(B_s^0 \rightarrow f_0 J/\psi)}. \quad (59)$$

We use data from CDF and D0 for the  $B_s^0 \rightarrow \phi J/\psi$  decay, whereas there is no available data on the channel  $B_s^0 \rightarrow f_0 J/\psi$ . Our data compilation consists of the D0 values for the amplitudes,  $|\mathcal{A}_L|^2 = 0.555 \pm 0.027 \pm 0.006$ ,  $|\mathcal{A}_\parallel|^2 = 0.244 \pm 0.032 \pm 0.014$  and the relative phase  $\phi_\parallel = 2.72_{-0.27}^{+1.12}$  rad [7]. The CDF values [33] are compatible,  $|\mathcal{A}_L|^2 = 0.530 \pm 0.021 \pm 0.007$  and  $|\mathcal{A}_\parallel|^2 = 0.230 \pm 0.027 \pm 0.009$ , and the PDG data book quotes the branching fraction,  $\mathcal{B}(B_s^0 \rightarrow J/\psi \phi) = (9.3 \pm 3.3) \times 10^{-4}$  [18].

The ratio  $\mathcal{R}_{f_0/\phi}$  has been argued [8] to be of the order  $0.2 - 0.3$ , based on the knowledge of the experimental

ratio of decay rates [34],

$$\frac{\Gamma(D_s^+ \rightarrow f_0 \pi^+ \rightarrow K^+ K^- \pi^-)}{\Gamma(D_s^+ \rightarrow \phi \pi^+ \rightarrow K^+ K^- \pi^-)} = 0.28 \pm 0.12, \quad (60)$$

and an estimate of the semileptonic, integrated branching fraction ratio  $\mathcal{B}(D_s^+ \rightarrow f_0 e^+ \nu, f_0 \rightarrow \pi^+ \pi^-) / \mathcal{B}(D_s^+ \rightarrow \phi e^+ \nu, \phi \rightarrow K^+ K^-) = (13 \pm 4)\%$  from CLEO [35]. The ratio  $\mathcal{R}_{f_0/\phi}$  was reassessed in terms of the differential decay ratio [36],

$$\begin{aligned} \mathcal{R}_{f_0/\phi} &= \frac{\frac{d\Gamma}{dq^2}(D_s^+ \rightarrow f_0 e^+ \nu, f_0 \rightarrow \pi^+ \pi^-)|_{q^2=0}}{\frac{d\Gamma}{dq^2}(D_s^+ \rightarrow \phi e^+ \nu, \phi \rightarrow K^+ K^-)|_{q^2=0}} \\ &= 0.42 \pm 0.11. \end{aligned} \quad (61)$$

If we combine the above three experimental estimates, we propose a window of  $0.2 \lesssim \mathcal{R}_{f_0/\phi} \lesssim 0.5$  for the ratio based on  $D_s$  decays.

With the experimental data listed under Eq. (59) as constraint, we find optimal values for  $X_C$  and  $X_H$ . In principle, we deal with a system of four coupled non-linear equations for  $|\mathcal{A}_L|^2$ ,  $|\mathcal{A}_\parallel|^2$ ,  $\phi_\parallel$  and  $\mathcal{B}(B_s^0 \rightarrow \phi J/\psi)$  and four variables, which does not put tight constraints on the phenomenological part of our  $B_s^0 \rightarrow J/\psi \phi$  amplitude. When solving numerically we find, depending on the  $f_{B_s^0}$  values, two solutions among which only

TABLE VIII: Same as Table VII but for the  $B_s^0 \rightarrow J/\psi f_0(980)$  amplitude.

$f_{B_s^0}$ [MeV]	230	260	290
$\zeta$	$-44.08 + i 4.81$	$-68.25 + i 4.59$	$-90.51 + i 6.40$

TABLE IX: Prediction for the  $B_s^0 \rightarrow J/\psi f_0$  observables for the different amplitudes  $LOVP$ ,  $LOVPH$  and  $LOVPH + C$  along with experimental analysis data of the  $B_s^0 \rightarrow J/\psi \phi$  decay. Here central values,  $f_{B_s^0} = 260$  MeV and  $\bar{f}_{f_0^s} = 380$  MeV, and the transition form factor  $F_1^{B_s^0 \rightarrow f_0^s}(q^2 = m_{J/\psi}^2) = 0.4$  are used. The values in the second column are predictions. Those of the third column include the hard scattering corrections with the endpoint parametrization  $\rho_H = 1.85 \pm 0.07$  and  $\phi_H = 255.9^\circ \pm 24.6^\circ$ . The fourth column corresponds to the reproduction of the data with the parameters  $\rho_H$ ,  $\phi_H$ ,  $\rho_C$  and  $\phi_C$  displayed in the second line of Table II.

	$LOVP$ (Prediction)	$LOVPH$ (2 parameters)	$LOVPH + C$ (4 parameters)	Experimental data
$ \mathcal{A}_L ^2$	0.172	0.554	0.555	$0.555 \pm 0.033$ [7]
$ \mathcal{A}_\parallel ^2$	0.404	0.219	0.244	$0.244 \pm 0.046$ [7]
$\phi_\parallel$ (rad)	-0.221	2.13	2.72	$2.72 \pm 1.38$ [7]
$\mathcal{B}(B_s^0 \rightarrow J/\psi \phi)$	0.00075	0.00115	0.00093	$0.00093 \pm 0.00033$ [18]
$\mathcal{B}(B_s^0 \rightarrow J/\psi f_0)$	0.00020	0.00047	0.00050	
$A_{CP}(B_s^0 \rightarrow J/\psi f_0)$	-0.00013	-0.0013	-0.0011	
$\mathcal{R}_{f_0/\phi}$	0.28	0.42	0.55	

one yields a reasonable value for the branching fraction  $\mathcal{B}(B_s \rightarrow f_0 J/\psi)$  not too different from that in a naive quark model. We list the parameters  $\rho_{C,H}$  and  $\phi_{C,H}$  independent of  $\bar{f}_{f_0^s}$  for three values of  $f_{B_s^0}$  in Table II, from which it is plain that the uncertainties on the magnitude of the modulus  $\rho_C$  as well as the phase  $\phi_C$  are substantial. The experimental errors on the observables are clearly not constraining enough. Yet, we observe that the variations of  $X_C$  and  $X_H$  are smooth as a function of the decay constant  $f_{B_s^0}$ .

Likewise, we present numerical values for  $a_n^{q,h}(m_b)$  for the three helicities in  $B_s^0 \rightarrow J/\psi \phi$  in Tables III, IV and V and for  $B_s^0 \rightarrow J/\psi f_0$  in Table VI as functions of  $f_{B_s^0}$  to illustrate one facet of the hadronic uncertainty. In these tables, we list the decomposition of  $a_n^{q,h}(m_b)$  for each value of  $f_{B_s^0}$ ; in the first column, the values of  $a_n^{q,h}(m_b)$  are for the calculated leading order (LO), vertex (V) and penguin (P) amplitudes only. These are independent of  $f_{B_s^0}$  and correspond to the predictions in Figure 1. Next, the  $a_n^{q,h}(m_b)$  that contain the LO, V, P and the hard-scattering (H) amplitudes, where only  $\rho_H$  and  $\phi_H$  are fitted to reproduce the  $B_s^0 \rightarrow \phi J/\psi$  observables while  $X_C = 0$ . For  $f_{B_s^0} = 260$  MeV one obtains  $\rho_H = 1.85 \pm 0.07$  and  $\phi_H = 255.9 \pm 24.6$ . These values are not very

different from those given in the second line of Table II. This case corresponds to Figure 2. At last, denoted by  $LOVPH + C$ , we give the values for  $a_n^{q,h}(m_b)$  for the case that the  $\zeta^{(h)}$  amplitudes are included, which corresponds to the  $\rho_{C,H}$  and  $\phi_{C,H}$  values in Table II and to Figure 3. We remind that the dependence on  $f_{B_s^0}$  enters the short-distance coefficients via the hard-scattering contribution  $H_n^h(M_1 J/\psi)$  in Eq. (22) and that the phenomenological amplitudes,  $X_H$  and  $X_C$ , are in competition with each other. Therefore, the hard scattering contributions to  $a_n^{q,h}(m_b)$  in  $LOVPH$  are slightly different than those to  $LOVPH + C$ .

The largest values observed in the leading amplitude,  $a_2^{u,c}(m_b)$ , are for  $h = 0$ . We also remark there is no variation as a function of  $f_{B_s}$  in Table IV since  $H_n^{h=-1}(M_1 J/\psi) = 0$ . Moreover, penguin contractions only contribute to  $a_7^{q,h=+1}(m_b)$  and  $a_9^{q,h=+1}(m_b)$  in the  $B_s^0 \rightarrow \phi J/\psi$  amplitudes, while there are no penguin terms in  $B_s^0 \rightarrow f_0 J/\psi$ . Altogether, the penguin contributions are very small. We note that the contribution of the phenomenological amplitudes,  $\zeta^{(h)}$  (Tables VII and VIII), is small, about 6–7% of the  $h = 0, +1$  amplitudes in  $B_s \rightarrow \phi J/\psi$  and 2% of the  $B_s \rightarrow f_0 J/\psi$  amplitude, yet dominant in the  $h = -1$  amplitude devoid of penguin and

hard scattering corrections. Thus, any contribution from new physics, and to less an extent annihilation topologies, should occur in the  $h = -1$  helicity amplitude.

When including all the contributions ( $LOVPH + C$ ), we qualitatively verify the hierarchy relation,  $|\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=0)}| > |\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=+1)}| > |\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=-1)}|$ , in  $B_s^0 \rightarrow J/\psi \phi$  and  $|\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=0)}| > |\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=-1)}| > |\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^{(h=+1)}|$  in the  $CP$  conjugate decay  $\bar{B}_s^0 \rightarrow J/\psi \phi$ . These hierarchy relations are also reproduced for the amplitudes when they include, besides tree contributions, vertex, penguin and hard-scattering corrections.

Having determined numerical values for  $X_H$  and  $X_C$ , we can calculate the  $B_s^0 \rightarrow f_0 J/\psi$  amplitude and obtain the associated branching fraction and  $CP$  asymmetry. We do so for the central values of  $f_{B_s^0} = 260$  MeV and  $\bar{f}_{f_0^s} = 380$  MeV discussed in Section V. For a transition form factor  $F_1^{B_s^0 \rightarrow f_0^s}(q^2 = m_{J/\psi}^2) = 0.4$  and for the different amplitudes  $LOVP$ ,  $LOVPH$  and  $LOVPH + C$  defined above, those observables are displayed in Table IX together with a comparison of the  $B_s^0 \rightarrow J/\psi \phi$  results with the corresponding available experimental analysis values. Furthermore, we obtain for a transition form factor  $F_1^{B_s^0 \rightarrow f_0^s}(q^2 = m_{J/\psi}^2) = 0.2$ :

$$\begin{aligned} \mathcal{B}(B_s \rightarrow f_0 J/\psi) &= 3.80 \times 10^{-4}, \\ A_{CP}(B_s \rightarrow f_0 J/\psi) &= -0.0005, \\ \mathcal{B}(B_s \rightarrow \phi J/\psi) &= 9.30 \times 10^{-4}, \\ \mathcal{R}_{f_0/\phi} &= 0.42; \end{aligned}$$

for  $F_1^{B_s^0 \rightarrow f_0^s}(q^2 = m_{J/\psi}^2) = 0.3$ ,

$$\begin{aligned} \mathcal{B}(B_s \rightarrow f_0 J/\psi) &= 4.37 \times 10^{-4}, \\ A_{CP}(B_s \rightarrow f_0 J/\psi) &= -0.0008, \\ \mathcal{B}(B_s \rightarrow \phi J/\psi) &= 9.30 \times 10^{-4}, \\ \mathcal{R}_{f_0/\phi} &= 0.48; \end{aligned}$$

and for  $F_1^{B_s^0 \rightarrow f_0^s}(q^2 = m_{J/\psi}^2) = 0.5$ ,

$$\begin{aligned} \mathcal{B}(B_s \rightarrow f_0 J/\psi) &= 5.7 \times 10^{-4}, \\ A_{CP}(B_s \rightarrow f_0 J/\psi) &= -0.0013, \\ \mathcal{B}(B_s \rightarrow \phi J/\psi) &= 9.30 \times 10^{-4}, \\ \mathcal{R}_{f_0/\phi} &= 0.63, \end{aligned}$$

and finally, the  $CP$  asymmetries in  $B_s \rightarrow J/\psi \phi$  are,

$$\begin{aligned} A_{CP}^L(B_s \rightarrow \phi J/\psi) &= -1.66 \times 10^{-3}, \\ A_{CP}^{\parallel}(B_s \rightarrow \phi J/\psi) &= 1.99 \times 10^{-3}, \\ A_{CP}^{\perp}(B_s \rightarrow \phi J/\psi) &= 2.15 \times 10^{-3}. \end{aligned}$$

Our prediction for the time-integrated asymmetry  $A_{CP}(B_s \rightarrow f_0 J/\psi)$  is about one order of magnitude smaller than the Standard Model value,  $-2\beta_s = -0.036$ .

We remark that the above numerical values for this  $CP$  asymmetry have to be interpreted with care — we choose the parameters of the full QCDF amplitude in Table II such that the experimental  $B_s^0 \rightarrow J/\psi \phi$  observables are reproduced. In doing so, we may deliberately include “new physics” effects with just the Standard Model amplitude, in particular via the additional amplitudes  $\zeta^{(h)}$ . Moreover, we use the same end-point parameterization,  $X_H$ , in both decay channels since the  $B_s^0 \rightarrow J/\psi f_0$  branching ratio is not experimentally known. This approach seems reasonable, as the physics buried in these infrared divergences must be similar in both decays. It could also lead to an overestimation of the hard-scattering contributions to  $B_s^0 \rightarrow J/\psi f_0$  as well as of  $A_{CP}(B_s \rightarrow f_0 J/\psi)$ .

We illustrate the variation of the ratio,  $\mathcal{R}_{f_0/\phi}$ , by taking into account the uncertainties in the decay constants  $f_{B_s^0}$  and  $\bar{f}_{f_0^s}$  as well as those in the decay rates,  $\mathcal{B}(f_0(980) \rightarrow \pi^+ \pi^-) = 0.50_{-0.09}^{+0.07}$  [10, 36] and  $\mathcal{B}(\phi \rightarrow K^+ K^-) = 0.489 \pm 0.005$  [18]. The results are displayed in Figures 1–3.

In Figure 1,  $\mathcal{R}_{f_0/\phi}$  is plotted as a function of  $F_1^{B_s^0 \rightarrow f_0^s}(m_{J/\psi}^2)$  where only the tree amplitude along with vertex and penguin corrections are included in both amplitudes,  $\mathcal{A}_{B_s^0 \rightarrow \phi J/\psi}^h$  and  $\mathcal{A}_{B_s^0 \rightarrow f_0 J/\psi}$ . The ratio is plotted with the corresponding envelope of  $\mathcal{R}_{f_0/\phi}$  due to the uncertainty on the decay rates. In Figure 2, we augment this amplitude by hard-scattering contributions, that is the full QCDF amplitude given in Eq. (22). Finally, in Figure 3,  $\mathcal{R}_{f_0/\phi}$  is plotted as a function of  $F_1^{B_s^0 \rightarrow f_0^s}(m_{J/\psi}^2)$  including hard-scattering corrections and possible “other” contributions,  $\zeta^{(h)}$ . Although the aforementioned uncertainties are depicted in all figures, we stress that those on the decay constants  $f_{B_s^0}$  and  $\bar{f}_{f_0^s}$ , where they apply, have more impact on the  $\mathcal{R}_{f_0/\phi}$  band than the  $f_0(980)$  and  $\phi$  decay rate uncertainties. The spreading of the curves representing  $\mathcal{R}_{f_0/\phi}$  as a function of  $F_1^{B_s^0 \rightarrow f_0^s}(m_{J/\psi}^2)$  is larger with respect to the variation in  $f_{B_s^0}$  than in  $\bar{f}_{f_0^s}$ . This points to the necessity of having an improved experimental determination of  $f_{B_s^0}$ . The upper limit of the envelope is reached only for the largest values of  $f_{B_s^0}$  and  $\bar{f}_{f_0^s}$  considered here.

Figure 3 shows that our central-value predictions of  $\mathcal{R}_{f_0/\phi}$ , in absence of any phenomenological contributions, are within the estimate by Stone and Zhang [8] for most values of the form factor  $F_1^{B_s^0 \rightarrow f_0^s}(m_{J/\psi}^2)$ . However, when the additional amplitudes,  $\zeta$ , are accounted for in the decay amplitudes of Eqs. (2) and (4), the ratio  $\mathcal{R}_{f_0/\phi}$  exhibits three striking features:

- Additional amplitudes,  $\zeta$ , can play a major role due to their large contributions to both the numerator and denominator of the ratio  $\mathcal{R}_{f_0/\phi}$ , as seen from the comparison of Figures 2 and 3.
- The predicted  $\mathcal{R}_{f_0/\phi}$  band overlaps well with the

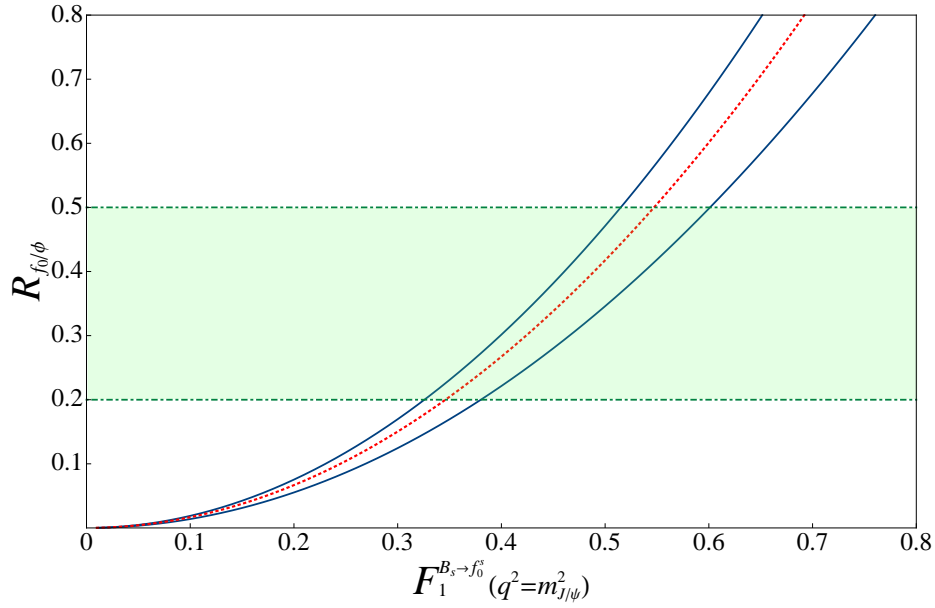


FIG. 1: The ratio  $\mathcal{R}_{f_0/\phi}$  as a function of the transition form factor  $F_1^{B_s^0 \rightarrow f_0^*}(m_{J/\psi}^2)$ . Only tree, vertex and penguin contributions (*LOVP*), independent of the decay constants  $f_{B_s^0}$  and  $\bar{f}_{f_0}$ , are included in the decay amplitudes. The dotted line corresponds to the central value of this ratio while the area between the two solid lines gives its envelope due to the uncertainties on the decay rates  $f_0(980) \rightarrow \pi^+\pi^-$  [10, 36] and  $\phi \rightarrow K^+K^-$  [18]. The two horizontal dash-dotted lines delimit the (shaded) area between the experimental predictions found in Refs. [8] and [36].

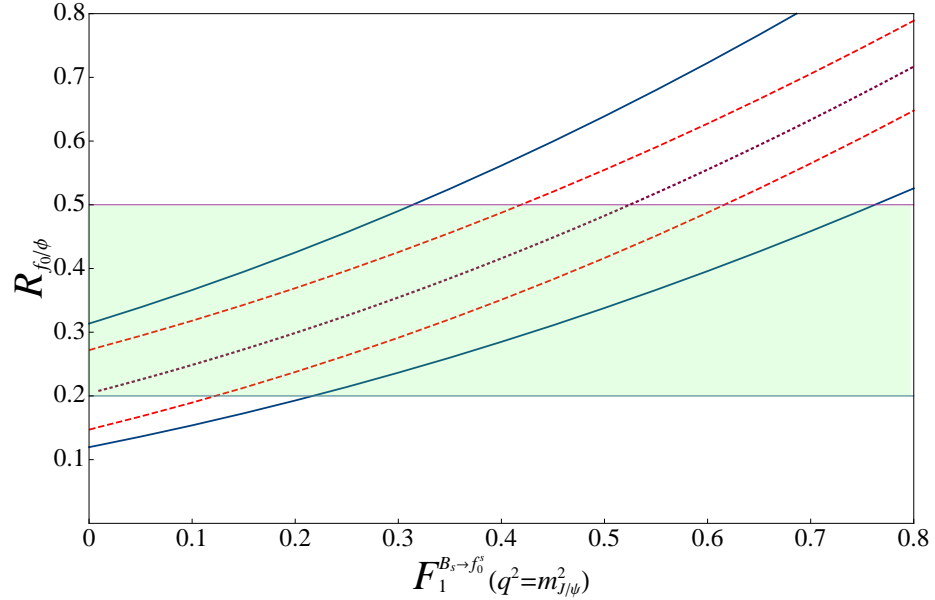


FIG. 2: The ratio  $\mathcal{R}_{f_0/\phi}$  as a function of the transition form factor  $F_1^{B_s^0 \rightarrow f_0^*}(m_{J/\psi}^2)$  where now the tree, vertex, penguin, and hard-scattering contributions (*LOVPH*) are included. The area between the two dashed lines gives the envelope of this ratio when taking into account uncertainties on the decay constants ( $f_{B_s^0} = 260 \pm 30$  MeV and  $\bar{f}_{f_0} = 380 \pm 40$  MeV) while the solid lines include in addition the uncertainties on the decay rates  $f_0(980) \rightarrow \pi^+\pi^-$  [10, 36] and  $\phi \rightarrow K^+K^-$  [18]. The single dotted line is our prediction for the central values of the decay constants,  $f_{B_s^0} = 260$  MeV and  $\bar{f}_{f_0} = 380$  MeV. The horizontal dash-dotted lines correspond to the experimental predictions of Refs. [8] and [36].

estimates of Refs. [8] and [36] for  $F_1^{B_s^0 \rightarrow f_0^*}(m_{J/\psi}^2) < 0.4$ ; beyond, our predictions are much larger, which

may indicate a larger pollution due to  $f_0(980) \rightarrow K^+K^-$  if contributions from other than the Standard Model were present.

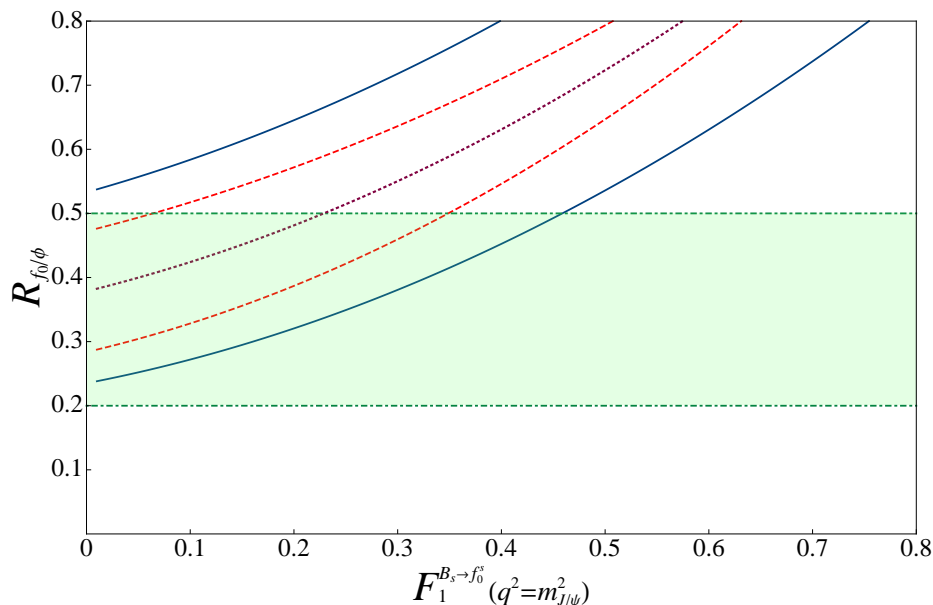


FIG. 3: Same as in Fig. 2 but including  $\zeta^{(h)}$  contributions ( $LOVPH + C$  amplitudes).

- The uncertainties on the  $f_0(980)$  and  $\phi$  decay rates increase the width of the band considerably, though the main uncertainty stems from the decay constants  $f_{B_s^0}$  and  $f_{f_0}$ .

Let us remind that the decay constant  $\bar{f}_{f_0^s}$  only enters the hard-scattering and additional phenomenological contributions ( $C$ ) to the decay amplitude  $B_s^0 \rightarrow f_0(980)J/\psi$ . If these are turned off, as in Figure 1, the ratio  $R_{f_0/\phi}$  is still significantly above 10% for realistic transition-form factor values. That said, for practical purposes we decide to only consider the more recently obtained decay constants in Ref. [22] and choose three values within the given errors,  $\bar{f}_{f_0^s} = 340, 380, 420$  MeV. The value 180 MeV [25] yields too low branching fractions in other decays, for example  $B \rightarrow f_0(980)\pi, f_0(980)\rho, f_0(980)K^{(*)}$ . Nevertheless, since we fix the hard-scattering parameters,  $\rho_H$  and  $\phi_H$ , only via the decay  $B_s^0 \rightarrow J/\psi\phi$  and  $\bar{f}_{f_0^s}$  enters the numerator in  $\mathcal{R}_{f_0/\phi}$  linearly, the modification is straightforward:  $\bar{f}_{f_0^s} = 180$  MeV is about half the value  $\bar{f}_{f_0^s} = 380$  MeV, therefore the central value of  $\mathcal{R}_{f_0/\phi}$  in Figure 2 decreases from 0.42 to 0.19 (for  $F_0^{B_s^0 \rightarrow f_0^s} = 0.4$ ). This is still within the limits predicted by the experimental estimates,  $0.2 \lesssim \mathcal{R}_{f_0/\phi} \lesssim 0.5$ , and implies an  $S$ -wave pollution.

We infer from our numerical results that  $S$ -wave kaons or pions under the  $\phi$  peak in  $B_s^0 \rightarrow J/\psi\phi$  are very likely to originate from the similar decay  $B_s^0 \rightarrow J/\psi f_0$ . Therefore, the extraction of the mixing phase,  $-2\beta_s$ , from  $B_s^0 \rightarrow J/\psi\phi$  may well be biased by this  $S$ -wave effect which should be taken into account in experimental analyses. In our interpretation of the full QCDF amplitude, we not only confirm the influence of  $S$ -wave contamination

as advocated in Refs. [8] and [36] but also find that its effect could be sizable.

## VII. CONCLUSIVE OUTLOOK

The “phase” of  $B_s^0 - \bar{B}_s^0$  mixing,  $-2\beta_s$ , is thought to be best measured in the golden decay,  $B_s^0 \rightarrow J/\psi\phi$ , and provides an opportune place to investigate physics beyond the Standard Model. Several models have been proposed to explain the apparent discrepancy of the Standard Model prediction for  $-2\beta_s$  with recent experiments, in particular exploring the impact of heavy, as of yet undiscovered particles on  $CP$  violation in weak  $B$ -meson decays. A general analysis of possible new physics effects in the case of  $B_s^0 - \bar{B}_s^0$  mixing was recently given by Chiang *et al.* [21]. In there, the authors investigate several beyond Standard Model variations of the  $B_s \rightarrow J/\psi\phi$  decay, such as  $Z^{(\prime)}$ -mediated Flavor Changing Neutral Currents (FCNC), two Higgs doublets and SUSY, and find that new physics contributions may only modestly contribute to the mixing phase. However, it is also concluded, somewhat prematurely, that the CDF and  $D0$  results are clear signs of new physics.

In the present paper, we have taken a different path and studied the contamination of final state  $S$ -waves kaons in the  $B_s^0 \rightarrow J/\psi\phi$  channel by those originating from the  $f_0(980)$  in the very similar  $B_s^0 \rightarrow J/\psi f_0(980)$  decay. We find that this effect is strong enough already for amplitudes including leading order, vertex and Penguin corrections to create a real bias in the determination of  $-2\beta_s$ .

Of course, we are aware that the phenomenological endpoint parametrization of  $\alpha_s$  corrections in the am-

plitudes  $H_n(M_1 J/\psi)$  and  $H_n^h(M_1 J/\psi)$  can cloud possible new physics contributions alongside the  $\zeta^{(h)}$  contributions. In this case, we suppose that any new effects should be of comparable magnitude in  $B_s^0 \rightarrow J/\psi\phi$  and  $B_s^0 \rightarrow J/\psi f_0(980)$ . Therefore, the  $S$ -wave contamination would be on the upper side of the estimate we propound and future analyses of the mixing angle in  $B_s$  decays should be concerned with this effect.

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